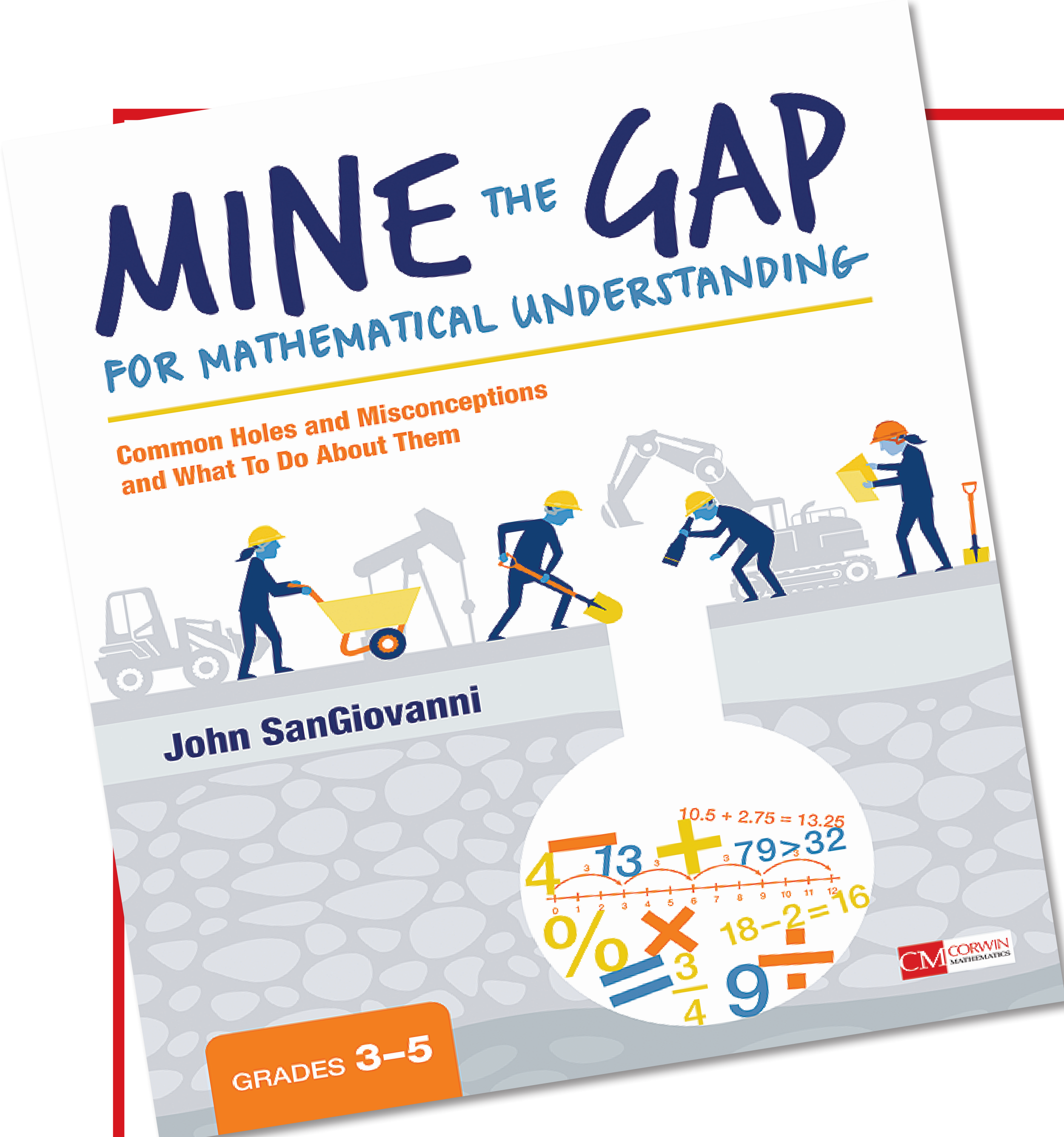

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CORWIN

BIG IDEA

43

BIG IDEA 43

Problem Solving With Multiplication and Division of Fractions

TASK 43A

A bakery uses $2\frac{1}{2}$ cups of sugar for a cake.

The chart shows the number of cakes they make each day of the week.

How many more cups of sugar do they need on Friday than on Tuesday?

Use pictures, numbers, or words to explain your thinking.

Days	Cakes Made
Sunday	10
Monday	4
Tuesday	5
Wednesday	5
Thursday	6
Friday	7
Saturday	10

About the Task

Making sense of fractions, especially computation with fractions, may be best done in context. This collection of tasks provides multiplication of fractions in context. This first task is not a traditional story problem. It replicates the types of problems we are more likely to encounter in the real world. In this problem, students have to gather data from a table, calculate with it, and compare the results.

PAUSE AND REFLECT

- **How does this task compare to tasks I've used?**
- **What might my students do in this task?**

Visit this book's companion website at **resources.corwin.com/minethegap/3-5** for complete, downloadable versions of all tasks.

Anticipating Student Responses

Students will identify how they found the values for Friday and Tuesday. Some students will multiply each day by $2\frac{1}{2}$ before finding the difference. Other students may find the difference of cakes made between the 2 days and then multiply that amount by $2\frac{1}{2}$. Both approaches are reasonable, although the latter could be considered more efficient. Some of our students may make calculation errors. Students may convert $2\frac{1}{2}$ to an improper fraction before computing, while others will find partial products by multiplying by 2 and then by $\frac{1}{2}$. Errors may occur because students don't understand what the problem is asking, because a key word is lacking, because there are multiple steps, or because they misread the table.

NOTES

WHAT THEY DID

Student 1

Student 1's work shows significant misunderstanding. Her work represents two different challenges students might have when working with this problem. She subtracts $2\frac{1}{2}$ from 10 because she notes that the problem asks "*how many more*" and she connects this phrase with subtraction. Her computation is also flawed. She says $\frac{10}{1}-2=2\frac{9}{1}$. To make sense of this error, we can presume that she subtracts in either direction to find a result. In other words, she subtract left to right for the numerator (10 – 1). She subtracts right to left for the denominator (2 – 1). It's likely that she simply brings the whole number over to her "difference."

MINING TIP

Avoid using key words as an instructional approach to problem solving. Doing so can create misconceptions about problems and set students up for incorrect solutions.

Student 2

Student 2 uses the more efficient strategy. She finds the difference of the 2 days (9 – 5). She then multiplies the difference by $2\frac{1}{2}$. She uses this strategy instead of multiplying both values by $2\frac{1}{2}$ and then finding the difference of the products. Her note about the "key" communicates that each number in the table is multiplied by $2\frac{1}{2}$. She doesn't multiply $4\times2\frac{1}{2}$ correctly. Instead, she multiplies 4 × 2 and then adds the half to the product, yielding an incorrect result of $8\frac{1}{2}$.

USING EVIDENCE

What would we want to ask these students? What might we do next?

Student 1

This problem shows that Student 1 needs work with problem solving and computation. We can work on both concepts at the same time. First, we want to develop her understanding of problem solving by developing problem-solving strategies. We can have her work with models and drawings to represent models and connect these with equations. Our work with computation should revisit addition and subtraction of fractions. We would be wise to begin with fractions less than 1 before moving to mixed numbers. It would also be wise to work with less complicated denominators, including halves, fourths, eighths, and twelfths.

MINING HAZARD

We must consider all of students' work when determining their understanding. We can combine their written thoughts with their diagrams or drawings to establish full understanding. However, additional ideas may not always link their ideas.

Student 2

Student 2 shows proficiency with solving the problem. She has made sense of the problem and applies an efficient strategy. *Her writing doesn't fully explain why she subtracted 9 – 5. Looking above, we may think she is linking the values to the associated days. But after looking closely, we see that she is making bonds of 10, which is not relevant to the task.* Our next step with Student 2 is to revisit multiplication with fractions and mixed numbers. It may be a simple oversight that she did not include the half in her multiplication. But, it may be a more significant problem.

TASK 43A: A bakery uses $2\frac{1}{2}$ cups of sugar for a cake. The chart shows the number of cakes they make each day of the week. How many more cups of sugar do they need on Friday than on Tuesday? Use pictures, numbers, or words to explain your thinking.

Student Work 1

A bakery uses 2½ cups of sugar for a cake.

The chart shows the number of cakes they make each day of the week.

Days	Cakes Made
Sunday	10
Monday	4
Tuesday	5
Wednesday	5
Thursday	6
Friday	9
Saturday	10

How many more cups of sugar do they need on Friday than on Tuesday?

$\frac{10}{1} - 2\frac{1}{2} = 11$

Use pictures, numbers, or words to explain your thinking.

I did $\frac{10}{1} - 2\frac{1}{2} = 2\frac{9}{1}$ I subtracted because the problem says how many more cups of sugar and $2\frac{9}{1}$ is an improper fraction $9+2=11$ and that is how I got my answer.

Student Work 2

A bakery uses 2½ cups of sugar for a cake.

The chart shows the number of cakes they make each day of the week.

Days	Cakes Made
Sunday	10
Monday	4
Tuesday	5
Wednesday	5
Thursday	6
Friday	9
Saturday	10

How many more cups of sugar do they need on Friday than on Tuesday?

$9-5=4 \times 2\frac{1}{2} = 8\frac{1}{2}$ Key $2\frac{1}{2}$

Use pictures, numbers, or words to explain your thinking.

I Know they awnser because nine Subtrack Five equal four, four times toward onehafe equal eight and $\frac{1}{2}$ I think.

WHAT THEY DID

Student 3

Student 3's strategy is logical. She identifies the number of cakes made on both days. She multiplies these values by $2\frac{1}{2}$. She then subtracts the products to find a difference. Her strategy is sound. Her solution is incorrect. She multiplies each number (9 and 5) by $2\frac{1}{2}$ incorrectly. In each, she multiplies the whole numbers and ignores the half. She then adds the half back to the product.

MINING HAZARD

Students who don't provide their reasoning in sentences or paragraphs may be discounted. It's important to remember that drawings, diagrams, and equations can fully communicate understanding. This is especially true when equations are labeled.

Student 4

Student 4 completes quite a bit of computation to find her solution. It is fine that she multiplies both days by $2\frac{1}{2}$ before finding the difference. *We can clearly see why she multiplied by 9 and 5, respectively, as she connects the products with the days*. We can also see how she found the products. She then shows her subtraction.

USING EVIDENCE

What would we want to ask these students? What might we do next?

Student 3

Like Student 2, Student 3 incorrectly multiplies with mixed numbers. She incorrectly applies a partial products strategy. That is, she breaks apart the mixed number into a whole number and a fractional part. She should then multiply both parts by the other factor. However, she only multiplies the whole numbers. It's also possible that she doesn't know how to multiply fractions or mixed numbers. Our first work is to investigate if she can multiply fractions by whole numbers. If so, we can then move to multiplying mixed numbers by whole numbers. We can reestablish partial products of mixed numbers and connect that to multiplication with whole numbers and fractions. As always, we want to make use of models and pictures connected to equations. Another strategy is to convert mixed numbers into improper fractions before multiplying. The strategy may be a complicated process that our students struggle to make sense of. A student who shows challenges with multiplying a mixed number such as $2\frac{1}{2}$ by a whole number may have considerable challenges with the conversion strategy. We must be careful to avoid overemphasis of procedure without conceptual understanding in this case.

Student 4

Student 4 does well. However, she does a lot of computation. Her work is full of procedure. This isn't problematic. But Student 4 might benefit from working with or discussing other strategies. She may find that her procedures for solving problems can be simplified. During discussion, she may find that she could find the difference of the days before multiplying (Student 2). She may also find that she can decompose the mixed number before multiplying so that the computation is less complicated (Students 2 and 3).

TASK 43A: A bakery uses $2\frac{1}{2}$ cups of sugar for a cake. The chart shows the number of cakes they make each day of the week. How many more cups of sugar do they need on Friday than on Tuesday? Use pictures, numbers, or words to explain your thinking.

Student Work 3

A bakery uses 2½ cups of sugar for a cake.

The chart shows the number of cakes they make each day of the week.

Days	Cakes Made
Sunday	10
Monday	4
Tuesday	5
Wednesday	5
Thursday	6
Friday	9
Saturday	10

How many more cups of sugar do they need on Friday than on Tuesday?

2½x9=18½

2½x5=10½

18½
−10½
8 cups of sugar

Use pictures, numbers, or words to explain your thinking.

I got 8 wholes because it says How many more did Friday than tuesday. So I did 2½ multiplied the days that where in the question and I got 18½ and 10½ so I subtracted those numbers and got my answer witch is 8 cups of sugar.

Student Work 4

A bakery uses 2½ cups of sugar for a cake.

The chart shows the number of cakes they make each day of the week.

Days	Cakes Made
Sunday	10
Monday	4
Tuesday	5
Wednesday	5
Thursday	6
Friday	9
Saturday	10

How many more cups of sugar do they need on Friday than on Tuesday?

2½=1 cake

2½x5 $\frac{5}{1}\times\frac{1}{2}=\frac{5}{2}=2\frac{1}{2}$ 2½+10=12½ cups on tuesday

2x5=10

2½x9 $\frac{1}{2}\times\frac{9}{1}=9/2=4\frac{1}{2}$ 2x9=18

18+4½=22½ cups on friday

Use pictures, numbers, or words to explain your thinking.

22½
−12½
10 cups of suger more

OTHER TASKS

- What will count as evidence of understanding?
- What misconceptions might you find?
- What will you do or how will you respond?

Visit this book's companion website at **resources.corwin.com/minethegap/3-5** for complete, downloadable versions of all tasks.

TASK 43B: A construction company is paving 18 miles of road. It can pave $\frac{3}{4}$ of a mile in a day. How many days will it take to complete the road? Use models, numbers, or words to explain your answer. The company says it will take 3 more days to complete a total of 20 miles. Do you agree or disagree with the company?

Task 43B is a problem with two different prompts. In the first prompt, students find the quotient of $18 \div \frac{3}{4}$. Some students may simply note that $\frac{3}{4}$ is about 1, so the company can do about 1 mile per day. With this thinking, students will justify 18. Others may justify that it is a little more or a little less than $\frac{3}{4}$ as they attempt to compensate for the fact that $\frac{3}{4}$ is not one whole. Successful students will find their solution with equations or drawings. Some students may multiply thinking of an unknown factor while others may multiply because they don't understand the context of the problem. In the second prompt, students have to reason about extending the road being paved. In this extension, the road is extended by 2 miles but 3 additional days are needed because of the rate of paving.

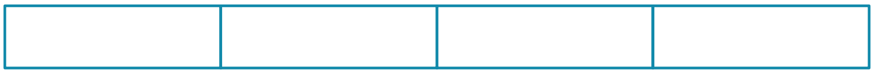
TASK 43C: Solve each problem. Use models, numbers, or words to justify your solution.

> **Jordan needs to collect 28 stickers for a prize. He has collected $\frac{3}{4}$ of the stickers. How many stickers has he collected?**
>
> **A restaurant uses $\frac{1}{2}$ pound of dough to make a loaf of bread. The restaurant has $8\frac{3}{4}$ pounds of dough for bread. How many loaves can the restaurant make?**

The word problems in this task make use of multiplication and division. The first problem generates a whole number as the solution. The second problem does not. In that problem, students will need to make sense of the $\frac{1}{4}$ of a pound left over. In both problems, equations, drawings, or other models such as bar diagrams may be used to find a solution. In the second problem, students may build up to $8\frac{3}{4}$ by skip-counting by halves. In a similar fashion, students may quickly make sense that there are two halves in one whole so there are 16 halves in eight wholes before making sense of the last $\frac{3}{4}$. Students may be unsuccessful for many reasons. The fraction computation may be a challenge. We can replace the fractions with whole numbers to identify if the fractions contribute to the problem. Other students may have difficulty making sense of the problems or identifying a strategy for solving. We want to ask questions to help students understand the context of the problem and the relationship between the numbers. This has greater long-term impact than having these students identify important information and look for words that indicate what operation they may use.

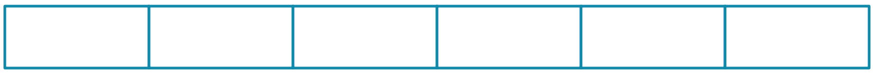
TASK 43D: **Complete each model to represent the division expression.**

$4 \div \frac{1}{4} =$



$5 \div \frac{1}{2} =$

$6 \div \frac{2}{3} =$



Write a word problem for one of the expressions above.

Writing word problems for expressions or equations supports the development of problem-solving skills. When students do this, they need to think about context, how problems are posed, how numbers relate to one another, and how questions in problems may be asked. We often do this with addition, subtraction, multiplication, or division of whole numbers. Applying this idea to multiplication or division with fractions can be considerably more challenging. Providing the models before the story supports the work in two ways. It helps us see if students can represent the division expression with a model, and it helps students make sense of the expression so that they can wrap a context around it.

NOTES