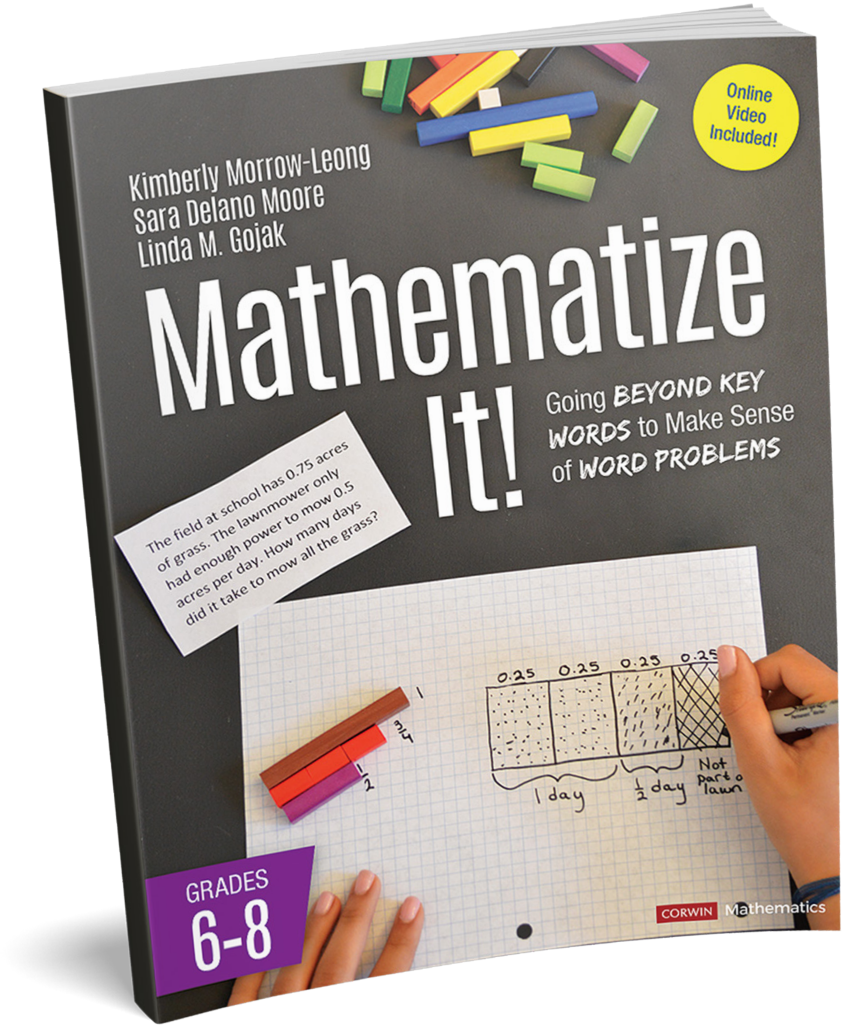


Thank you

FOR YOUR INTEREST IN CORWIN

Please enjoy this complimentary excerpt from *Mathematize It! Grades 6-8* by Kimberly Morrow-Leong, Sara Delano Moore, and Linda M. Gojak.

LEARN MORE about this title, including Features, Table of Contents, and Reviews.

CHAPTER TWO

Add-To and Take-From

Thinking About Active Addition and Subtraction

In this chapter we introduce two addition and subtraction problem situations, Add-To and Take-From. These are what we call active situations, a designation that will become more meaningful as you work through this chapter and the next, explore the characteristics of these problem types, and make comparisons with other problem types that show relationships. You'll notice that these situations are labeled Result Unknown, Change Addend Unknown, and Start Addend Unknown in the problem situation table. After we do some mathematics, we will return to a discussion of these variations. Let's get started.

Pretend you are walking into a workshop. This is the first of eight workshops exploring problem situations that all students will encounter in grades 6–8. In this workshop, as in the other seven, you will be exploring in detail aspects of these problems and the operations associated with them that you may not have considered before. Doing so requires that you take on the role of student, see the problems with new eyes, and let yourself try out representations and models for yourself. Because most middle school standards do not address or distinguish between the different problem contexts, each of the workshops will ask you to suspend your most efficient solution strategies, if only briefly, in order to see word problems and their contexts in a new light. In this chapter we are interested in how students (and you!) might represent a problem context using a model with fidelity to the context.

Mathematize It! Going Beyond Key Words to Make Sense of Word Problems, Grades 6–8 by Kimberly Morrow-Leong, Sara Delano Moore, and Linda M. Gojak. Copyright © 2021 by Corwin Press, Inc. All rights reserved.

Addition and Subtraction Problem Situations

	Result Unknown	Change Addend Unknown	Start Addend Unknown	
Add-To	Paulo paid $4.53 for his sandwich. Then he added $1.50 for a carton of milk to finish his lunch. How much was his lunch? $4.53 + 1.5 = x$ $4.53 = x - 1.5$	Paulo paid $4.53 for the sandwich in his lunch. Then he added a carton of milk to his tray to finish his lunch. The total for his lunch is $6.03. How much is a carton of milk? $4.53 + x = 6.03$ $4.53 = 6.03 - x$	Paulo added a sandwich to his tray. He added a carton of milk that cost $1.50 to his tray. With the sandwich and milk, his lunch cost $6.03. How much does the sandwich cost? $x + 1.5 = 6.03$ $6.03 - 1.5 = x$	
Take-From	There are 186 students in the 7th grade. 35 left to get ready to play in the band at the assembly. How many students are not in the band? $186 - 35 = x$ $35 + x = 186$	There are 186 students in the 7th grade. After the band students left class for the assembly, there were 151 students still in their classrooms. How many students are in the band? $186 - x = 151$ $x + 151 = 186$	35 band students left class to get ready to play in the assembly. There were 151 students left in the classrooms. How many students are in the 7th grade? $x - 35 = 151$ $35 + 151 = x$	

ACTIVE SITUATIONS

	Result Unknown	Change Addend Unknown	Start Addend Unknown	
Add-To	Paulo paid $4.53 for his sandwich. Then he added $1.50 for a carton of milk to finish his lunch. How much was his lunch? $4.53 + 1.5 = x$ $4.53 = x - 1.5$	Paulo paid $4.53 for the sandwich in his lunch. Then he added a carton of milk to his tray to finish his lunch. The total for his lunch is $6.03. How much is a carton of milk? $4.53 + x = 6.03$ $4.53 = 6.03 - x$	Paulo added a sandwich to his tray. He added a carton of milk that cost $1.50 to his tray. With the sandwich and milk, his lunch cost $6.03. How much does the sandwich cost? $x + 1.5 = 6.03$ $6.03 - 1.5 = x$	
Take-From	There are 186 students in the 7th grade. 35 left to get ready to play in the band at the assembly. How many students are not in the band? $186 - 35 = x$ $35 + x = 186$	There are 186 students in the 7th grade. After the band students left class for the assembly, there were 151 students still in their classrooms. How many students are in the band? $186 - x = 151$ $x + 151 = 186$	35 band students left class to get ready to play in the assembly. There were 151 students left in the classrooms. How many students are in the 7th grade? $x - 35 = 151$ $35 + 151 = x$	

RELATIONSHIP (NONACTIVE) SITUATIONS

	Total Unknown	One Part Unknown		Both Parts Unknown
Part-Part-Whole	The local ice cream shop asked customers to vote for their favorite new flavor of ice cream. 119 customers preferred mint chocolate chip ice cream. 37 preferred açai berry ice cream. How many customers voted? $119 + 37 = x$ $x - 119 = 37$	The local ice cream shop asked customers which new ice cream flavor they like best. 156 customers voted. 37 customers preferred açai berry ice cream. The rest voted for mint chocolate chip ice cream. How many customers voted for mint chocolate chip ice cream? $37 + x = 156$ $x = 156 - 37$		The local ice cream shop held a vote for their favorite new flavor of ice cream. The options were mint chocolate chip and açai berry ice cream. What are some possible combinations of votes? $x + y = 156$ $156 - x = y$
	Difference Unknown	Greater Quantity Unknown	Lesser Quantity Unknown	
Additive Comparison	Jessie and Roberto both collect baseball cards. Roberto has 53 cards and Jessie has 71 cards. How many fewer cards does Roberto have than Jessie? $53 + x = 71$ $53 = 71 - x$	Jessie and Roberto both collect baseball cards. Roberto has 53 cards and Jessie has 18 more cards than Roberto. How many baseball cards does Jessie have? $53 + 18 = x$ $x - 18 = 53$	Jessie and Roberto both collect baseball cards. Jessie has 71 cards and Roberto has 18 fewer cards than Jessie. How many baseball cards does Roberto have? $71 - 18 = x$ $x + 18 = 71$	

Note: These representations for the problem situations in this table reflect our understanding based on a number of resources. These include the tables in the Common Core State Standards for mathematics (CCSS-M; National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010); the problem situations as described in the Cognitively Guided Instruction research (Carpenter, Hiebert, & Moser, 1981), in Heller and Greeno (1979), and in Riley, Greeno, and Heller (1984); and other tools. See the Appendix and the companion website for a more detailed summary of the documents that informed our development of these tables.

Mathematize It! Going Beyond Key Words to Make Sense of Word Problems, Grades 6–8 by Kimberly Morrow-Leong, Sara Delano Moore, and Linda M. Gojak. Copyright © 2021 by Corwin Press, Inc. All rights reserved.

Sandbox Notes

As you enter into the Explore phase of problem solving, gather your tools, including markers or colored pencils, base 10 blocks, place value disks, two-color counters, and any other tools that you routinely have available in your classroom. Try several of the concrete manipulatives and some hand-drawn picture models in order to reflect the **mathematical story** in the word problem. If you put the problem in your own words, revisit your restatement and specify where you can see each quantity in the problem, and in the models you have created. Think about how your work can express your understanding of the problem situation.

Ask yourself these questions to focus your thinking:

- *Think about the quantities in each situation. What do they represent? What action is taking place between the quantities in the problem?*
- *How can you represent the quantities in the word problems with your manipulatives or pictures? Feel free to use the workspace provided.*
- *What equation best shows what is happening in each story?*

Mathematical story: A retelling of the action or relationships in a word problem or other problem context in a way that highlights the important mathematical details over any other information.

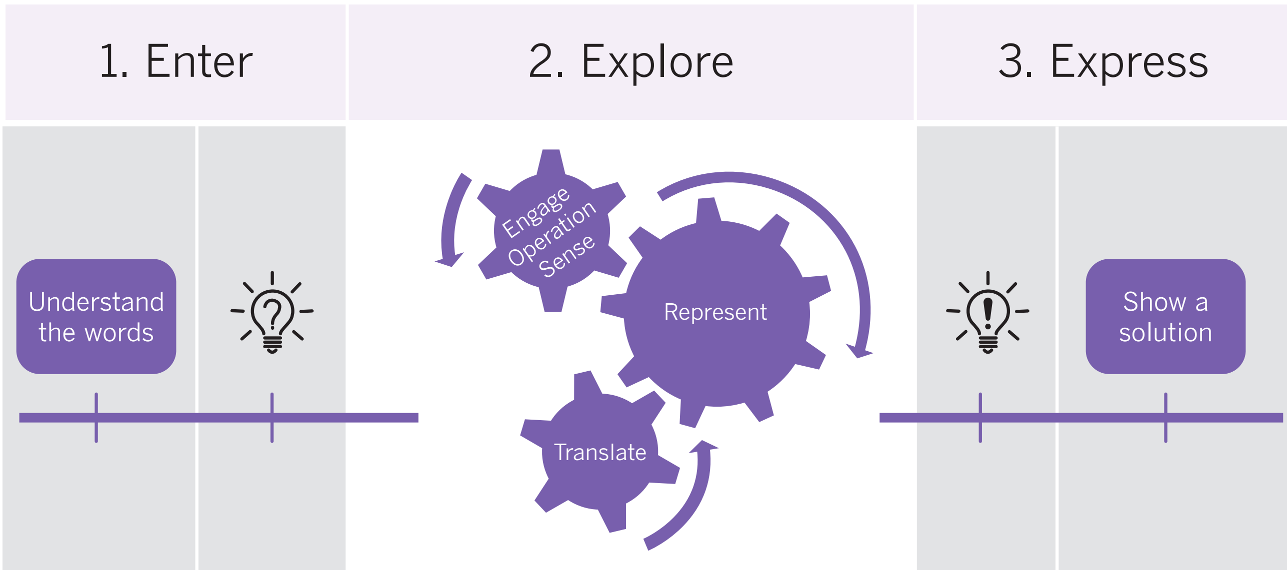
To begin, read the two problems in Figures 2.1 and 2.2. Don't try to solve them just yet. Instead, put yourself in the place of your students, and as you enter each problem, focus on understanding the words in each one. Use your own words to rephrase the problem, without focusing on the quantities. If necessary, substitute the quantities with the word *some*. This will help you curb your instinct to jump to a solution path before you fully explore the problem. Space is given below for recording your restatement of the problem. Look at the sandbox model in Figure 2.3 to remind yourself of how these tasks fit into the problem-solving process. You are now entering the mathematizing sandbox!

FIGURE 2.1	FIGURE 2.2
Emily is crocheting a scarf for her grandmother. She crocheted $2\frac{1}{2}$ feet over the winter break and then she added another $1\frac{1}{4}$ feet in January. How long is the scarf at the end of January?	Jim is collecting cans for recycling and weighs his total each week. He forgot to weigh the cans he collected last week, but this week he added another $1\frac{1}{4}$ pounds of cans to his collection, and he now has a total of $3\frac{3}{4}$ pounds ready to take to the recycling center. How much did the cans he collected last week weigh?

ENTER THE PROBLEM		

Mathematize It! Going Beyond Key Words to Make Sense of Word Problems, Grades 6–8 by Kimberly Morrow-Leong, Sara Delano Moore, and Linda M. Gojak. Copyright © 2021 by Corwin Press, Inc. All rights reserved.

FIGURE 2.3 A MODEL FOR MATHEMATIZING WORD PROBLEMS



To explore, you may need to take notes on your explorations on scratch paper. Once you can answer these questions, you are ready to show and justify your solutions. Include these in the workspace provided so that you can easily refer back to them. If your solution includes a concrete model, reproduce it clearly in a drawing. Be sure to add a verbal representation of the problem or additional notes on your thinking so that your solution is clear. *Remember: When the focus is on mathematizing, finding a solution is not the same as finding the answer. A solution is a representation of the problem that reveals how it can be solved. The answer comes after.* Jumping immediately to an efficient equation or finding an answer through mental math may be efficient, but it isn't a model for learning to recognize the general problem situation type. This idea will become clearer as you work through problems and explore the work samples of students and teachers throughout the book.

EXPLORE		

Mathematize It! Going Beyond Key Words to Make Sense of Word Problems, Grades 6–8 by Kimberly Morrow-Leong, Sara Delano Moore, and Linda M. Gojak. Copyright © 2021 by Corwin Press, Inc. All rights reserved.

Look back at your first trip to the mathematizing sandbox. Make sure you've translated the story of each problem into your own words and explored several different concrete, pictorial, and symbolic representations of the problem situations. You have likely translated your representations into an equation that you can then solve, but the answer may also be evident in another representation of the problem. You have probably used your operation sense to approach these word problems, and we hope that in the mathematizing sandbox you thought about ways that students might have the opportunity to show (or build) a deeper understanding of the problem situation rather than simply computing an answer.

STUDENTS AND TEACHERS THINK ABOUT THE PROBLEMS

Look at the student work in Figures 2.4 and 2.5 and consider how these students describe or draw what is happening in the related word problems. Look at the teacher commentary that follows and consider what the teachers noticed in the student work. There is also a video available exploring each student's solution to the problem, which you may want to watch once you have read through the student work and teacher comments.

What do you notice? Did you notice that there is action in both of these problems? Emily is crocheting her scarf while Jim is collecting cans. This action of *creating* a scarf or *collecting* cans signals addition. These are both examples of an **Add-To** problem situation. We want to highlight an important feature of Add-To problem situations: They all require the action of joining quantities within the context of the story, but as the comment in Figure 2.5 suggests, Add-To problem situations do not necessarily require addition in a solution. As we explore the nature of Add-To and Take-From problem situations, the distinction between the computation used to arrive at the answer and the operation associated with the problem situation will become increasingly clear.

Add-To: A problem situation that includes action happening in the problem: Some quantity is being added to the original quantity.

FINDING THE UNKNOWN, THREE STORY STRUCTURES

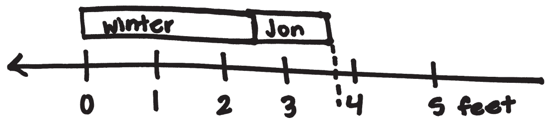
These active mathematical stories have a narrative structure—a beginning, a middle, and an end. In the beginning there is a starting value. Emily had no scarf when she started crocheting, and then she crocheted $2\frac{1}{2}$ feet after winter break and added another $1\frac{1}{4}$ feet in January. Jim collected some recycling each week, and his action of adding more cans is shown by adding more fraction bars. In each case the middle of the story arc brings a change in quantity: more scarf is made, more cans collected. The end of the story is the result of the change that took place. The scarf got longer and the weight of the collection of aluminum cans grew. In most word problems, only one of these three quantities is unknown. The table shown at the beginning of the chapter is organized to highlight each of these quantities in turn. For example, the unknown quantity could be the start of the problem situation, the change in the situation, or the result of the change, each reflecting the arc of the storyline. Identifying the unknown—in other words, the information we need to know—is the first step in any problem-solving process.

RESULT UNKNOWN The Result Unknown version of the Add-To problem situation is the most common in addition or subtraction word problem situations. As a matter of fact, students first learn to solve the Result Unknown variation of these problems, where we know the starting value and the change, in kindergarten (McCallum, Daro, & Zimba, n.d.). The scarf problem in Figure 2.1 is an example of an Add-To, Result Unknown problem situation. Note that the values in this problem are more appropriate for students in fifth grade, but since middle school students still need practice with fraction computation, we decided to include these values as examples in problems.

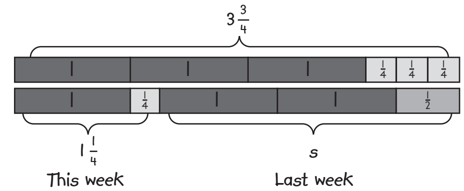
Mathematize It! Going Beyond Key Words to Make Sense of Word Problems, Grades 6–8 by Kimberly Morrow-Leong, Sara Delano Moore, and Linda M. Gojak. Copyright © 2021 by Corwin Press, Inc. All rights reserved.

FIGURE 2.4 | **FIGURE 2.5**

STUDENT WORK



$2\frac{1}{2} + 1\frac{1}{4} = 3\frac{3}{4}$



$1\frac{1}{4} + s = 2\frac{1}{2}$

TEACHER RESPONSE

This student told me he drew a picture of the scarf over a number line because he imagined using a tape measure to see how much Emily has finished. I was impressed that he didn't have trouble finding a common denominator-so many students still struggle with fractions. I wonder if the number line helped him with that.

I was surprised to see this student use an addition approach to this problem because it felt like subtraction to me. I can see from the labels on the lower bars how she might have figured it out with addition because the two main parts are labeled "this week" and "last week," so I can see both weeks of can collections. I see $1\frac{1}{4}$ first, and the second set shows how the student might have thought about finding the answer by counting on: adding on 1 and another 1 and finally a $\frac{1}{2}$.

VIDEO

Video 2.1

Crocheting a Scarf With a Number Line

To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.

Video 2.2

Collecting Recycling With Fraction Bars

Emily is crocheting a scarf for her grandmother. She crocheted $2\frac{1}{2}$ *feet over the winter break and then added another* $1\frac{1}{4}$ *feet in January. How long is the scarf at the end of January?*

Crochet during winter break + crochet in January = a scarf

$$2\frac{1}{2}+1\frac{1}{4}=x$$

Mathematize It! Going Beyond Key Words to Make Sense of Word Problems, Grades 6–8 by Kimberly Morrow-Leong, Sara Delano Moore, and Linda M. Gojak. Copyright © 2021 by Corwin Press, Inc. All rights reserved.

CHANGE UNKNOWN The next variation that students explore is Change Unknown. Students are likely to come up with strategies for decoding these problem situations later than for the Result Unknown variety. In the case of the Change Unknown variety of problem situation, there is a beginning and an end to the story (the starting and resulting values) and the student has to figure out what happened in the middle—what change led from the start to the result? Here is the scarf problem, rewritten to be a change unknown variety:

Emily is crocheting a scarf for her grandmother. She crocheted $2\frac{1}{2}$ *feet over the winter break and then added some more in January. At the end of January, the scarf is* $3\frac{3}{4}$ *feet long. How much did Emily crochet during January?*

Crochet during winter break + crochet in January = a scarf

$$2\frac{1}{2} + x = 3\frac{3}{4}$$

You may notice that the verbal equation does not change from one version of the scarf problem situation to the other. This is because the problem context itself does not change. Only the value that is missing changes.

START UNKNOWN The Start Unknown variation is the most challenging for students. It often requires students to work backward from the end of the story, through the change, to figure out the starting value (the beginning of the story). Because this requires more abstract reasoning, students are often expected to have mastered this final variation (with grade-appropriate values) after the others. Figure 2.2 illustrates the Start Unknown variation of an Add-To problem situation. Again, note that the translation of the problem situation into words represents the overall problem situation, no matter which value is missing. Despite the fact that students will typically master this variation of the Add-To problem situation last, it should be included in problem collections so that students have more opportunities to familiarize themselves with the problem structure and gain experience solving this type of problem.

Jim is collecting cans for recycling. He forgot to weigh the cans he collected last week, but this week he added $1\frac{1}{4}$ *pounds of cans to his collection, and he has a total of* $3\frac{3}{4}$ *pounds ready to take to the recycling center. How much did the cans he collected last week weigh?*

Cans collected last week + cans collected this week = all cans

$$x + 1\frac{1}{4} = 3\frac{3}{4}$$

STORY STRUCTURES: IMPLICATIONS FOR TEACHING

When we ask teachers to compose Add-To word problems, almost 75 percent of those problems are of the Result Unknown variation. It's no wonder that teachers and students sometimes feel like Change Unknown or Start Unknown problems are trick questions! Without being aware of it, teachers may be ignoring these kinds of problems to their students' detriment. Skipping these variations not only limits students' ability to grasp the dynamics of these stories, but it also limits their ability to transfer their thinking to other, more complex problem situations. As you'll see in the examples that follow, it is easy to adjust a problem from one variation to another, and it is critically important to expose students to a broad range of problems. By making sure your students have experience with all three variations, you will better prepare them to recognize Add-To situations in any form.

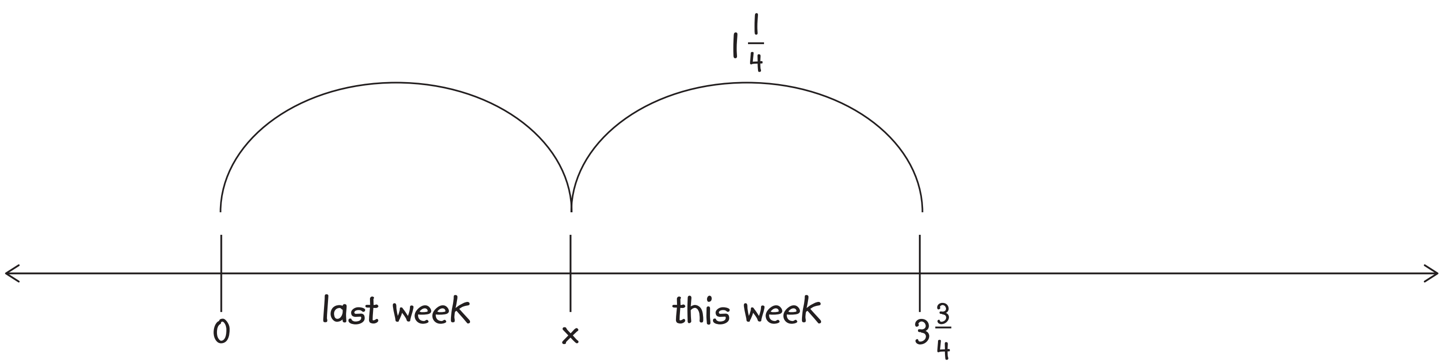
Your students in grades 6–8 should be familiar with all three variations of Add-To problems, but they might not be equally confident with all forms. If this is the case, it is especially important that you provide them with opportunities to experience all varieties! You don't have to start from scratch, but introducing the problem situations with familiar values will free your class to focus more on the structure of the problem than on the numbers themselves.

Mathematize It! Going Beyond Key Words to Make Sense of Word Problems, Grades 6–8 by Kimberly Morrow-Leong, Sara Delano Moore, and Linda M. Gojak. Copyright © 2021 by Corwin Press, Inc. All rights reserved.

As students in middle grades develop their number sense about rational numbers and integers, the underlying structure of the problem situations does not change. Their strategies for modeling and finding solutions, however, can change to be more practical for the numbers in the problem. They may move from marked number lines to open number lines or use two-color counters to represent the problem situation when integers are involved. It will be helpful to have a solid foundation in the structures as you work with your students on the more complex number categories of integers and negative rational numbers described later in this chapter.

Let's look again at the two problems we started with in this chapter, this time considering their different problem structures. The crochet problem (Figure 2.1) is an Add-To, Result Unknown variation, the one we are most familiar with, whereas Jim's recycling problem (Figure 2.2) is an Add-To, Start Unknown problem situation. It can be difficult to model a Start Unknown story because the beginning quantity is unknown. In this case the student appears to have used knowledge of the commutative property, beginning with the change quantity and then representing the unknown starting value. An open number line might have allowed the student to represent the unknown starting value first (see Figure 2.6).

FIGURE 2.6 USING AN OPEN NUMBER LINE TO REPRESENT AN UNKNOWN STARTING VALUE



As we saw in the teacher's response, a subtraction operation is often used to compute an answer to this problem type, even though the problem still remains an Add-To variety. In Chapter 3 we will explore more of what happens when the problem situation does not match the operation used to find a solution, including connecting to inverse operations.

Mathematize It! Going Beyond Key Words to Make Sense of Word Problems, Grades 6–8 by Kimberly Morrow-Leong, Sara Delano Moore, and Linda M. Gojak. Copyright © 2021 by Corwin Press, Inc. All rights reserved.