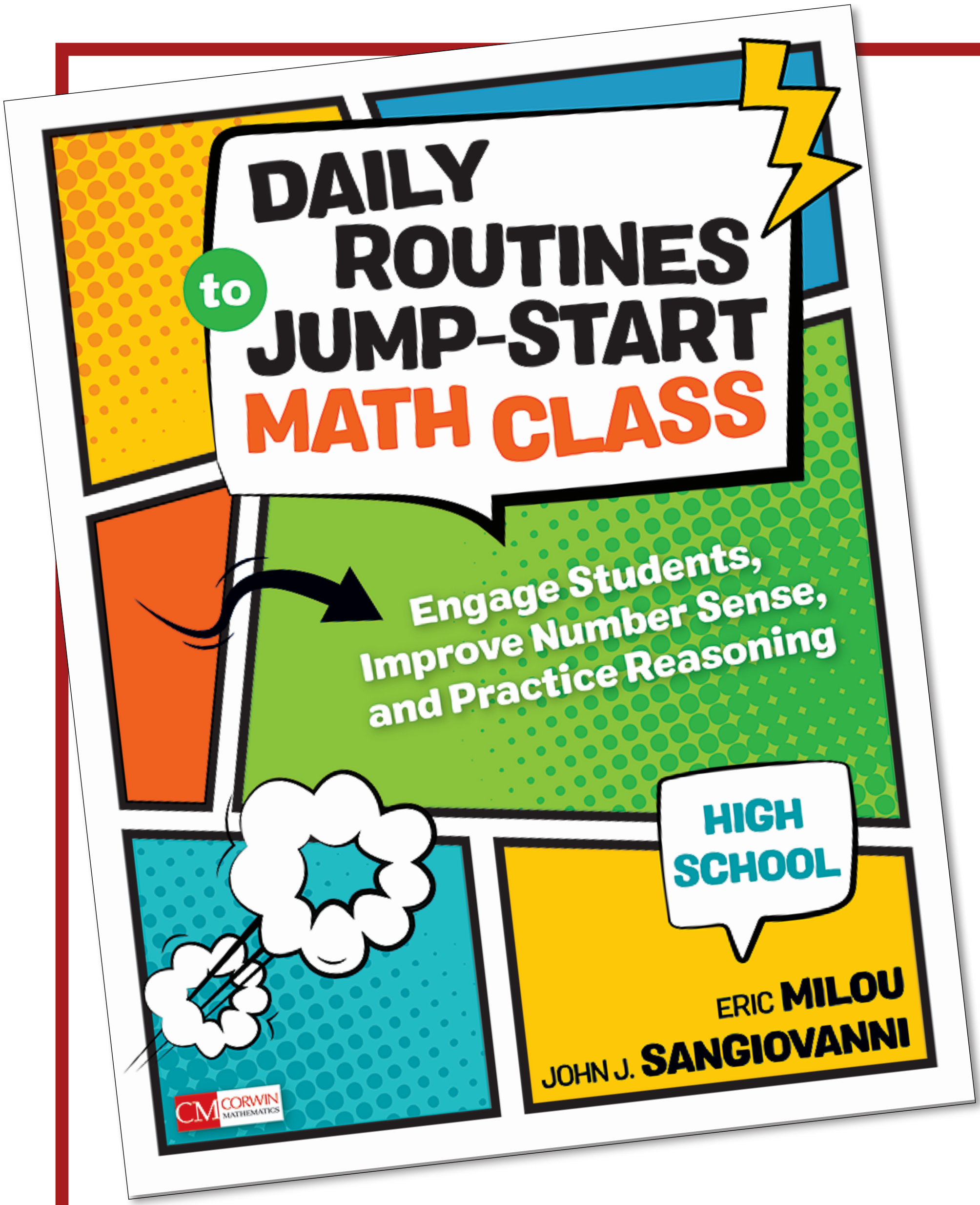


Thank you

FOR YOUR
INTEREST IN
CORWIN

Please enjoy this complimentary excerpt from *Daily Routines to Jump-Start Math Class, High School,* by Eric Milou and John J. SanGiovanni. This lesson allows student to work through multiple mathematical arguments in order to identify the errors in the argument. Students will have the opportunity analyze these errors in order promote higher level thinking in their path to finding the correct argument.

LEARN MORE about this title, including Features, Table of Contents, and Reviews.

ROUTINE 4

TWO WRONGS AND A RIGHT

Write the equation of a line in point-slope form that passes through (2, 5) and (5, 11). $m=\frac{11-5}{5-2}=\frac{6}{3}=2$ $y-5=2(x-2)$	Write the equation of a line in point-slope form that passes through (2, 5) and (5, 11). $m=\frac{5-11}{5-2}=\frac{-6}{3}=-2$ $y-11=-2(x-2)$
Write the equation of a line in point-slope form that passes through (2, 5) and (5, 11). $m=\frac{5-11}{5-2}=\frac{-6}{3}=2$ $y-5=-2(x-2)$	Two **wrongs** and a **right**

ABOUT THE ROUTINE

In *Two Wrongs and a Right*, students analyze the work of three mathematical arguments of which two are wrong and one is correct. Students determine which argument is correct and detect the mathematical errors in the others. This daily routine has the potential to add considerable value to any classroom, as it provides students with repeated opportunities to develop their sense-making and argument-constructing/critiquing abilities.

WHY IT MATTERS

Why is it a good idea to have students analyze math errors?

- It promotes the use of several mathematical practices including Mathematical Practice 1 (make sense of problems and persevere in solving them), Mathematical Practice 2 (reason abstractly and quantitatively), and especially Mathematical Practice 3 (construct viable arguments and critique the reasoning of others).

online resources All tasks can be downloaded for your use at **resources.corwin.com/jumpstartroutines/highschool**

- It promotes higher level thinking. Error analysis is at the top of the higher level thinking skills hierarchy. It requires students to create, analyze, and even prove what is correct.
- It aids in conceptual understanding. When students can find errors in a process and explain it (that part is key), they are really showing a conceptual understanding of the skill or concept.
- It is a great test-taking strategy. Teaching students to find errors in thinking and algorithms is a perfect way to prepare them for multiple-choice math questions in an authentic manner. When they solve a math problem and are unable to locate the answer in the choice, they can look over their work and see if they can determine the error in their own math.

WHAT THEY SHOULD UNDERSTAND FIRST

Clearly, students should learn to identify their own mathematical errors and self-correct their mistakes. This routine gives students such practice and the ability to study mathematical errors to identify the cause of the incorrect answer. This ability to check for correctness is a big key to achieving math proficiency.

WHAT TO DO

1. Display the three mathematical arguments to the problem to the class.
2. Have students work in pairs to determine which of the three is correct and determine the error in the other two.
3. Facilitate student reflection and discussion.
4. Help students think about the *assumptions* behind each mistake, whether there are *fragments of correctness* in a given mistake, and how they might *help a classmate* who was prone to such a mistake.

ANTICIPATED STRATEGIES FOR THIS EXAMPLE

Students will likely analyze each step of the arguments presented as follows:

- Top left: The student calculated the slope correctly and used the point-slope formula correctly.
- Top right: The student calculated the slope incorrectly and used numbers from two different points in the point-slope formula.
- Bottom left: The student calculated the slope incorrectly.

Write the equation of a line in point-slope form that passes through (2, 5) and (5, 11). $m = \frac{11-5}{5-2} = \frac{6}{3} = 2$ $y - 5 = 2(x - 2)$	Write the equation of a line in point-slope form that passes through (2, 5) and (5, 11). $m = \frac{5-11}{5-2} = \frac{-6}{3} = -2$ $y - 11 = -2(x - 2)$
Write the equation of a line in point-slope form that passes through (2, 5) and (5, 11). $m = \frac{5-11}{5-2} = \frac{-6}{3} = 2$ $y - 5 = -2(x - 2)$	Two **wrongs** and a **right**

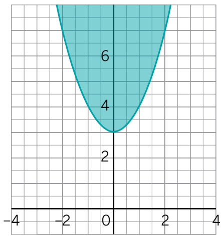
ADDITIONAL EXAMPLES

ALGEBRA

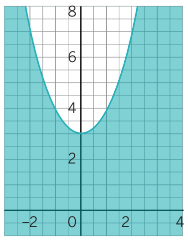
Examples 1 to 16 contain algebra content. Remind students that the same algebra problem or graph is presented three times, with one solution process correct but two incorrect. Ask students to find the correct solution and determine the incorrect steps and the reason for them in the others.

1

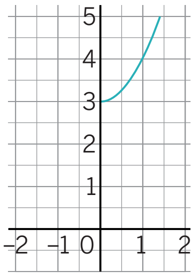
$y \geq x^2 + 3$



$y \geq x^2 + 3$



$y \geq x^2 + 3$



Two **wrongs** and a **right**

1.
- Top left: The student put the shading in the wrong location.
- Top right: Correct.
- Bottom left: The student graphed only the positive domain of the function (due to the ≥) instead of graphing the full function and shading correctly.

NOTES

2

Solve the multi-step equation: $4(-x-2)-5=13$ $-4x+8-5=13$ $-4x+3=13$ $-4x=10$ $x=\frac{-10}{4}=\frac{-5}{2}$	Solve the multi-step equation: $4(-x-2)-5=13$ $-4x-8-5=13$ $-4x-13=13$ $-4x=0$ $x=0$
Solve the multi-step equation: $4(-x-2)-5=13$ $-4x-8-5=13$ $-4x-13=13$ $-4x=26$ $x=\frac{-26}{4}=\frac{-13}{2}$	Two **wrongs** and a **right**

2.
- Top left: Distributed the 4 incorrectly in the first step.
- Top right: In the fourth line, did not add 13 to both sides.
- Bottom left: Correct.

3

Solve the absolute value inequality: $\|x + 5\| < 15$ $-15 < x + 5 < 15$ $-20 > x > 10$	Solve the absolute value inequality: $\|x + 5\| < 15$ $-15 < x + 5 < 15$ $-20 < x < 10$
Solve the absolute value inequality: $\|x + 5\| < 15$ $-15 < x + 5 < 15$ $-20 < x < 20$	Two **wrongs** and a **right**

3.
- Top left: In last step, when subtracting 5 from both sides, the inequality signs do NOT change.
- Top right: Correct.
- Bottom left: In last step, when subtracting 5 from both sides, incorrectly arrived at 20 (15 – 5 – 10).

4

Find the slope of the line that passes through (−2, 5) and (−2, 1). $m = \frac{5-1}{-2-2} = \frac{4}{-4} = -1$	Find the slope of the line that passes through (−2, 5) and (−2, 1). $m = \frac{5-1}{-2\ -\ -2} = \frac{4}{0}$ Slope is undefined
Find the slope of the line that passes through (−2, 5) and (−2, 1). $m = \frac{-2 - -2}{5-1} = \frac{0}{4} = 0$	Two **wrongs** and a **right**

4.

- Top left: The denominator is incorrect. Should be −2 − (−2).
- Top right: Correct.
- Bottom left: Subtracted the *x*-coordinates in the numerator and the *y*-coordinates in the denominator.

5

Given $f(x) = 5x - 2$, find the value of *x* when $f(x) = 8$. $f(8) = 5(8) - 2$ $f(8) = 38$	Given $f(x) = 5x - 2$, find the value of *x* when $\mathrm{f}(x) = 8$. $f(8) = 5(8) - 2$ $f(8) = 38$ $f = 4.75$
Given $f(x) = 5x - 2$, find the value of *x* when $f(x) = 8$. $8 = 5x - 2$ $10 = 5x$ $2 = x$	Two **wrongs** and a **right**

5.

- Top left: Incorrectly substituted 8 for *x*. We do not know the value of *x*. We only know that $f(x) = 8$.
- Top right: Solved for *f* in the last step; *f* is not a variable.
- Bottom left: Correct.

6

$(x-3)^2-5=11$ $(x-3)^2=16$ $x-3=4$ $x=7$	$(x-3)^2-5=11$ $(x-3)^2=16$ $x-3=\pm 4$ $x=3\pm 4$ $x=7$ or $x=-1$
$(x-3)^2-5=11$ $(x-3)^2=6$ $x-3=\pm 6$ $x=3\pm 6$ $x=9$ or $x=-3$	Two **wrongs** and a **right**

6.
- Top left: When taking the square root of both sides of an equation, you need to consider both the positive and negative root.
- Top right: Correct.
- Bottom left: First step incorrect; need to add 5 to both sides.

7

Simplify the expression: $\frac{2(3)(2x+6)}{6}$ $\frac{6(6x+18)}{6}$ $6x+18$	Simplify the expression: $\frac{2(3)(2x+6)}{6}$ $\frac{2(6x+18)}{6}$ $2(x+18)$ $2x+36$
Simplify the expression: $\frac{2(3)(2x+6)}{6}$ $\frac{6(2x+6)}{6}$ $2x+6$	Two **wrongs** and a **right**

7.
- Top left: Distributed the 3 twice.
- Top right: Incorrectly divided $\frac{(6x+18)}{6}$. It should yield $x + 3$.
- Bottom left: Correct.

8

$\frac{y^3x^{-2}}{y^{-4}x^3}$ $\frac{y^3y^4}{x^2x^3}$ $\frac{y^{12}}{x^6}$	$\frac{y^3x^{-2}}{y^{-4}x^3}$ $\frac{y^3y^4}{x^2x^3}$ $\frac{y^7}{x^5}$
$\frac{y^3x^{-2}}{y^{-4}x^3}$ $\frac{y^7}{x^1}$ $\frac{1}{x^1y^7}$	Two **wrongs** and a **right**

8.
- Top left: Incorrectly multiplied x^2x^3.
- Top right: Correct.
- Bottom left: Exponent of *x* variable is incorrect in first step. Student probably added the exponents 3 + (−2) instead of subtracting.

9

$\lvert -3-4\rvert + \lvert -6+1\rvert$ $\lvert -7\rvert + \lvert -5\rvert$ 7 + 5 12	$\lvert -3-4\rvert + \lvert -6+1\rvert$ $\lvert 3+4\rvert + \lvert 6+1\rvert$ 7 + 7 14
$\lvert -3-4\rvert + \lvert -6+1\rvert$ $\lvert -1\rvert + \lvert -5\rvert$ 1 + 5 6	Two **wrongs** and a **right**

9.
- Top left: Correct.
- Top right: Incorrect first step. Should first add/subtract the numbers inside the absolute values.
- Bottom left: $\lvert -3-4\rvert = \lvert -7\rvert$.

10

$x^2 - 5x + 6 = 2$ $(x - 3)(x - 2) = 2$ $x - 3 = 2$ or $x - 2 = 2$ $x = 5$ or $x = 4$	$x^2 - 5x + 6 = 2$ $x^2 - 5x + 4 = 0$ $(x + 4)(x + 1) = 0$ $x + 4 = 0$ or $x + 1 = 0$ $x = -4$ or $x = -1$
$x^2 - 5x + 6 = 2$ $x^2 - 5x + 4 = 0$ $(x - 4)(x - 1) = 0$ $x - 4 = 0$ or $x - 1 = 0$ $x = 4$ or $x = 1$	Two **wrongs** and a **right**

10.
- Top left: First step incorrect. Need to be sure that equation = 0.
- Top right: Factored incorrectly.
- Bottom left: Correct.

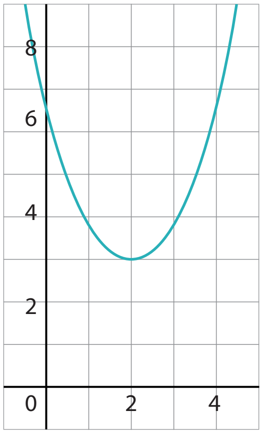
11

$\frac{x}{2}+\frac{x}{3}=6$ $3x+2x=36$ $5x=36$ $x=\frac{36}{5}$	$\frac{x}{2}+\frac{x}{3}=6$ $3x+2x=6$ $5x=6$ $x=\frac{6}{5}$
$\frac{x}{2}+\frac{x}{3}=6$ $\frac{3x}{6}+\frac{2x}{6}=6$ $\frac{5x}{6}=6$ $5x=36$ $x=\frac{5}{36}$	Two **wrongs** and a **right**

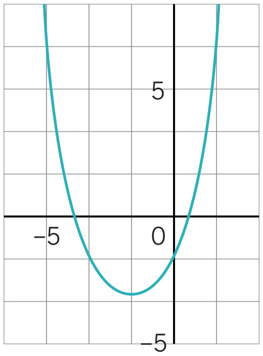
11.
- Top left: Correct.
- Top right: Did not correctly multiply right-hand side of equation by 6.
- Bottom left: Last step incorrect. Should have divided both sides by 5.

12

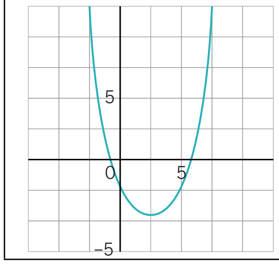
Parabola with vertex (2, −3)



Parabola with vertex (2, −3)



Parabola with vertex (2, −3)



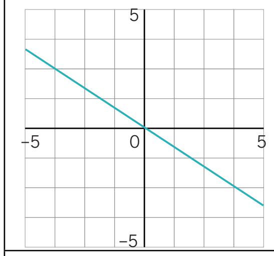
Two **wrongs** and a **right**

12.
- Top left: Vertex is (2, 3).
- Top right: Vertex is (−2, −3).
- Bottom left: Correct.

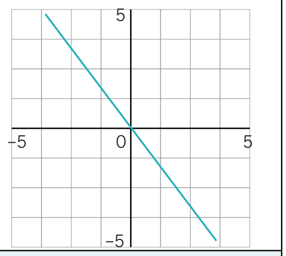
NOTES

13

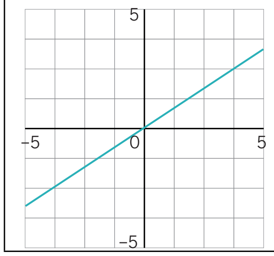
Line with slope $\frac{-2}{3}$



Line with slope $\frac{-2}{3}$



Line with slope $\frac{-2}{3}$



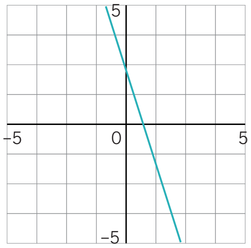
Two **wrongs** and a **right**

13.
- Top left: Correct.
- Top right: Slope is $\frac{-3}{2}$.
- Bottom left: Slope is $\frac{2}{3}$.

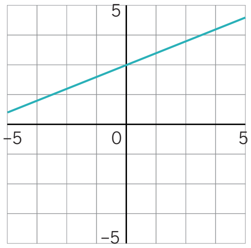
NOTES

14

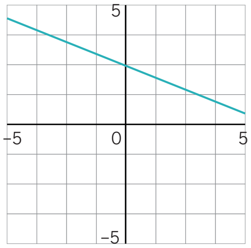
Line graphed is $\perp$ to $y = 3x+1$



Line graphed is $\perp$ to $y = 3x+1$



Line graphed is $\perp$ to $y = 3x+1$



Two **wrongs** and a **right**

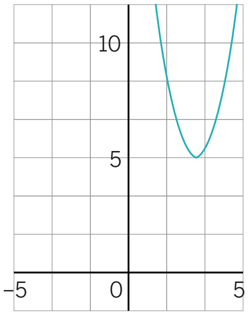
14.
- Top left: Slope is -3.
- Top right: Slope is $\frac{1}{3}$.
- Bottom left: Correct.

NOTES

15

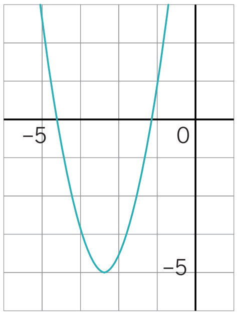
Graph of

$y = 2(x-3)^2 - 5$



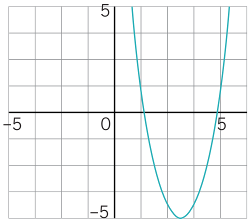
Graph of

$y = 2(x-3)^2 - 5$



Graph of

$y = 2(x-3)^2 - 5$



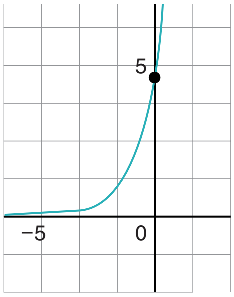
Two **wrongs** and a **right**

15.
- Top left: Graph of $y = 2(x-3)^2 + 5$.
- Top right: Graph of $y = 2(x+3)^2 - 5$.
- Bottom left: Correct.

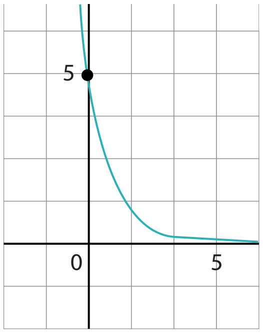
NOTES

16

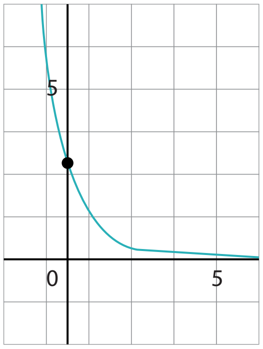
Graph of $y = 5\left(\frac{1}{3}\right)^x$



Graph of $y = 5\left(\frac{1}{3}\right)^x$



Graph of $y = 5\left(\frac{1}{3}\right)^x$



Two **wrongs** and a **right**

16.
- Top left: Graph of $y = 5(3)^x$.
- Top right: Correct.
- Bottom left: Graph of $y = 3\left(\frac{1}{3}\right)^x$.

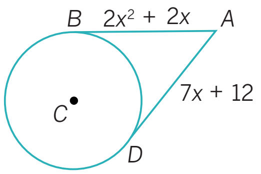
NOTES

GEOMETRY

The next set of examples (17–29) contains geometry content. The routine premise is the same, with two incorrect solutions and one correct.

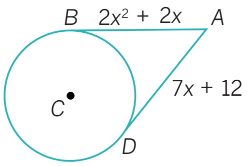
17

Points B and D are tangent to the circle.



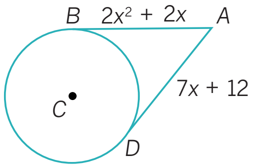
$2x^2 + 2x = 7x + 12$
$2x^2 - 5x - 12 = 0$
$(2x + 3)(x - 4) = 0$
$2x = -3$ or $x = 4$
$x = \frac{-3}{2}$ or $x = 4$

Points B and D are tangent to the circle.



$2x^2 + 2x = 7x + 12$
$2x^2 - 5x - 12 = 0$
$(2x + 3)(x - 4) = 0$
$2x = -3$ or $x = 4$
$x = 4$

Points B and D are tangent to the circle.



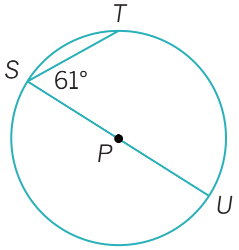
$2x^2 + 2x = 7x + 12$
$2x^2 - 5x - 12 = 0$
$(2x - 3)(x + 4) = 0$
$2x = 3$ or $x = -4$
$x = \frac{3}{2}$

Two **wrongs** and a **right**

17.
- Top left: Correct.
- Top right: Eliminated the negative solution, but in fact the solution is possible.
- Bottom left: Factored incorrectly.

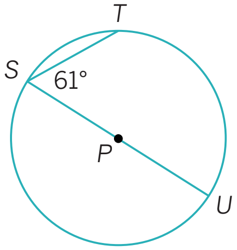
18

SU is the diameter of the circle. Find the $m\widehat{ST}$.



$m\widehat{ST} = 61$

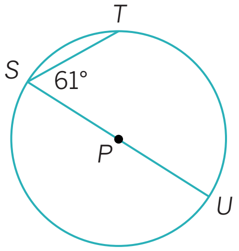
SU is the diameter of the circle. Find the $m\widehat{ST}$.



$m\widehat{TU} = 2(61) = 122$

$m\widehat{ST} = 122$

SU is the diameter of the circle. Find the $m\widehat{ST}$.



$m\widehat{TU} = 2(61) = 122$

$m\widehat{ST} = 180 - 122 = 58$

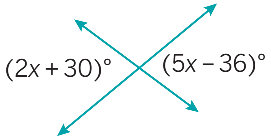
Two **wrongs** and a **right**

18.
- Top left: Incorrectly assumed that the angle TSU = $m\widehat{ST}$.
- Top right: Correctly found $m\widehat{TU}$, but $m\widehat{TU}$ does not equal $m\widehat{ST}$.
- Bottom left: Correct.

NOTES

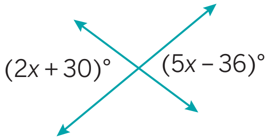
19

Solve for x.



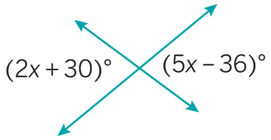
$$\begin{aligned} 2x+30 &= 5x-36 \\ 66 &= 3x \\ x &= 22 \end{aligned}$$

Solve for x.



$$\begin{aligned} 2x+30 = 5x-36 &= 180 \\ 7x-66 &= 180 \\ 7x &= 186 \\ x &= \frac{186}{7} \end{aligned}$$

Solve for x.



$$\begin{aligned} 2x+30+5x-36 &= 90 \\ 7x-6 &= 90 \\ 7x &= 96 \\ x &= \frac{96}{7} \end{aligned}$$

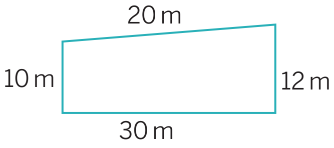
Two **wrongs** and a **right**

19.
- Top left: Correct.
- Top right: Incorrectly assumed that the angles were supplementary.
- Bottom left: Incorrectly assumed that the angles were complementary.

NOTES

20

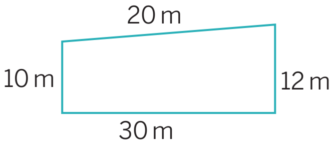
Find the area of the trapezoid.



$A = \frac{1}{2}(10 + 12)30$

$A = 330\ m^2$

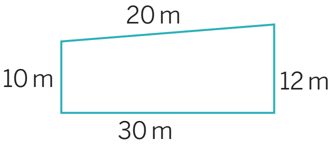
Find the area of the trapezoid.



$A = \frac{1}{2}(20 + 32)10$

$A = 250\ m^2$

Find the area of the trapezoid.



$A = \frac{1}{2}(10 + 12)(20 + 30)$

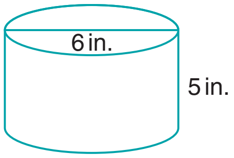
$A = 550\ m^2$

Two **wrongs** and a **right**

20.
- Top left: Correct.
- Top right: Incorrectly assumed the bases were 20 and 30.
- Bottom left: Did not identify the height correctly.

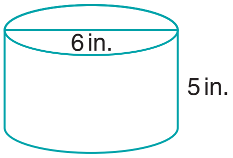
21

Find the volume of the cylinder.



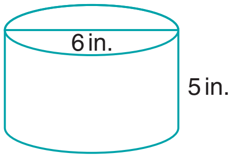
$V = \pi(5)^2 6 = 150\pi$

Find the volume of the cylinder.



$V = \pi(6)^2 5 = 180\pi$

Find the volume of the cylinder.



$V = \pi(3)^2 5 = 45\pi$

Two **wrongs** and a **right**

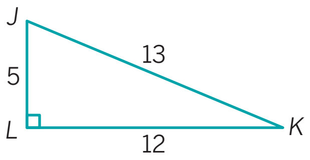
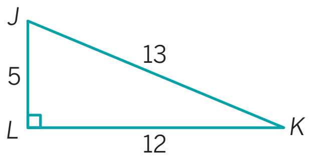
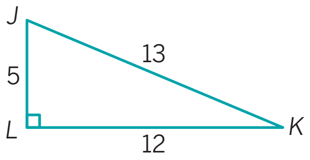
21.
- Top left: Radius is not 5.
- Top right: Radius is not 6.
- Bottom left: Correct.

22

Angles A and B are supplementary. $m\angle A = 36$; find $m\angle B$. $36 + m\angle B = 90$ $m\angle B = 54$	Angles A and B are supplementary. $m\angle A = 36$; find $m\angle B$. $36 + m\angle B = 180$ $m\angle B = 144$
Angles A and B are supplementary. $m\angle A = 36$; find $m\angle B$. $36 + m\angle B = 360$ $m\angle B = 324$	Two **wrongs** and a **right**

22.
- Top left: Used the definition of complementary angles.
- Top right: Correct.
- Bottom left: Used 360 instead of 180 degrees.

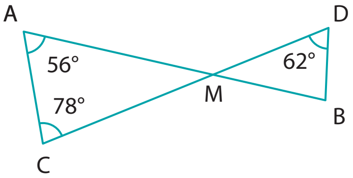
23

Find the Cos K  Cos K = $\frac{5}{12}$	Find the Cos K  Cos K = $\frac{12}{13}$
Find the Cos K  Cos K = $\frac{5}{13}$	Two **wrongs** and a **right**

23.
- Top left: Incorrectly used opposite/adjacent (which would be the Tan K).
- Top right: Correct.
- Bottom left: Incorrectly used opposite side/hypotenuse (which would be the Sin K).

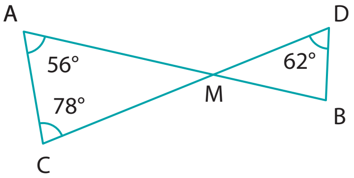
24

Find the $m\angle MBD$.



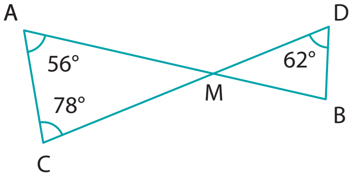
180 – (56 + 78)
180 – 134
46

Find the $m\angle MBD$.



Triangle MBD is isosceles, and thus, $m\angle MBD = m\angle MDB = 62$.

Find the $m\angle MBD$.



$m\angle AMC = 180 - (56 + 78)$

$m\angle AMC = 46$

$m\angle AMC = m\angle DMB = 46$

$m\angle MDB = 180 - (46 + 62)$

$m\angle MDB = 180 - (108)$

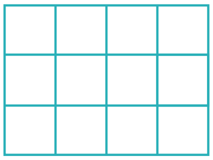
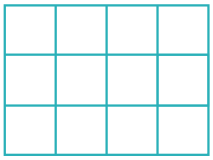
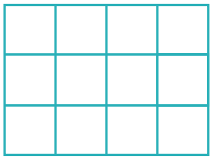
$m\angle MDB = 72$

Two **wrongs** and a **right**

24.
- Top left: Found the $m\angle AMC$.
- Top right: We do not know that Triangle MBD is isosceles.
- Bottom left: Correct.

NOTES

25

The rectangle has area = 432 and has 12 congruent squares. What is the perimeter?  Let the side of one of the small squares = *x*. Thus $(4x)(3x) = 432.$ $12x^2 = 432$ $x^2 = 36$ $x = 6$ $\text{Perimeter} = 6 \times 4 = 24$	The rectangle has area = 432 and has 12 congruent squares. What is the perimeter?  Let the side of one of the small squares = *x*. Thus $(4x)(3x) = 432.$ $12x^2 = 432$ $x^2 = 36$ $x = 6$ $\text{Perimeter} = 6 \times 14 = 84$
The rectangle has area = 432 and has 12 congruent squares. What is the perimeter?  It is NOT possible to find the perimeter.	Two **wrongs** and a **right**

25.
- Top left: The last step is incorrect as the length of the small square is 6.
- Top right: Correct.
- Bottom left: It is possible to find the perimeter.

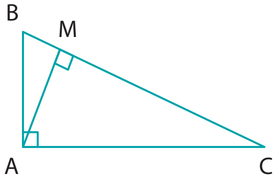
26

ABC is a right triangle.

$AM \perp BC$.

$m\angle ABC = 55$.

Find $m\angle MAC$.



$m\angle ABC + m\angle ACM + 90° = 180°$

$m\angle ACM = 180 - 90 - 55 = 35°$

$m\angle MAC + m\angle ACM + 90 = 180°$

$m\angle MAC = 180 - 90 - m\angle ACM$

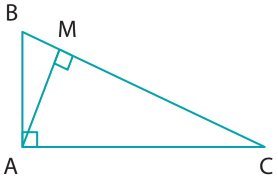
$= 180 - 90 - 55 = 35°$

ABC is a right triangle.

$AM \perp BC$.

$m\angle ABC = 55$.

Find $m\angle MAC$.



$m\angle ABC + m\angle ACM + 90 = 180°$

$m\angle ACM = 180 - 90 - 55 = 45°$

$m\angle MAC + m\angle ACM + 90 = 180°$

$m\angle MAC = 180 - 90 - m\angle ACM$

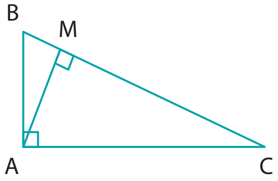
$= 108 - 90 - 45 = 45°$

ABC is a right triangle.

$AM \perp BC$.

$m\angle ABC = 55$.

Find $m\angle MAC$.



$m\angle ABC + m\angle ACM + 90 = 180°$

$m\angle ACM = 180 - 90 - 55 = 35°$

$m\angle MAC + m\angle ACM + 90 = 180°$

$m\angle MAC = 180 - 90 - m\angle ACM$

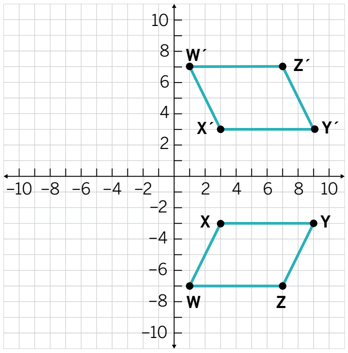
$= 180 - 90 - 35 = 55°$

Two **wrongs** and a **right**

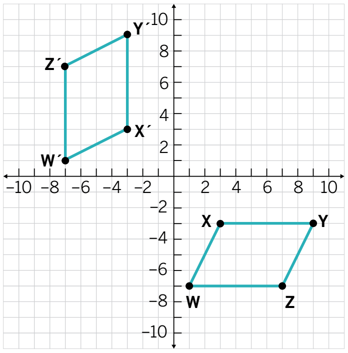
26.
- Top left: Substitution of 55 for $m\angle ACM$ is incorrect.
- Top right: Incorrect computation of 180 – 90 – 55.
- Bottom left: Correct.

27

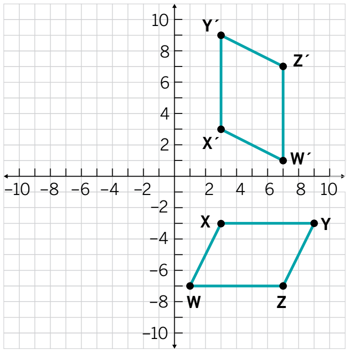
Parallelogram WXYZ is mapped to its image W′X′Y′Z′ by a reflection across the *x*-axis.



Parallelogram WXYZ is mapped to its image W′X′Y′Z′ by a reflection across the *x*-axis.



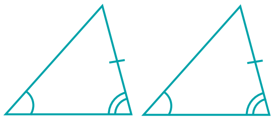
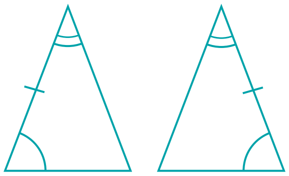
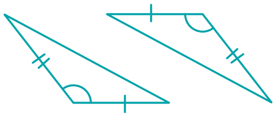
Parallelogram WXYZ is mapped to its image W′X′Y′Z′ by a reflection across the *x*-axis.



Two **wrongs** and a **right**

27.
- Top left: Correct.
- Top right: Shows a reflection across the line $y = x$.
- Bottom left: Shows a 90° counterclockwise rotation about the origin.

28

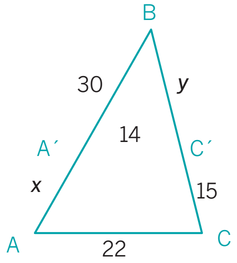
The following triangles are congruent by ASA. 	The following triangles are congruent by ASA. 
The following triangles are congruent by ASA. 	Two **wrongs** and a **right**

28.
- Top left: The triangles are congruent by AAS.
- Top right: Correct.
- Bottom left: The triangles are congruent by SAS.

NOTES

29

In $\triangle ABC$, $AC \parallel A'C'$. Find the length (y) of BC′.



$$\frac{22}{14} = \frac{y+15}{y}$$

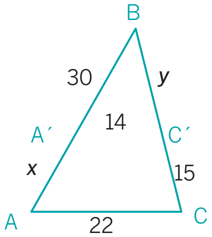
$$22y = 14(y+15)$$

$$22y = 14y + 210$$

$$8y = 210$$

$$y = 26.25$$

In $\triangle ABC$, $AC \parallel A'C'$. Find the length (y) of BC′.



$$\frac{14}{30} = \frac{y}{y+15}$$

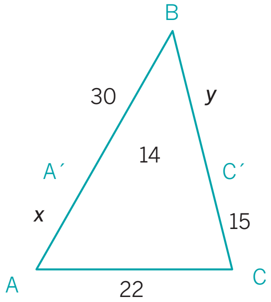
$$30y = 14(y+15)$$

$$30y = 14y + 210$$

$$16y = 210$$

$$y = 13.125$$

In $\triangle ABC$, $AC \parallel A'C'$. Find the length (y) of BC′.



$$\frac{22}{14} = \frac{15}{y}$$

$$22y = 210$$

$$y = 9.54$$

Two **wrongs** and a **right**

29.
- Top left: Correct.
- Top right: Incorrect proportion.
- Bottom left: Incorrect proportion.